Ratio Test:

Example 1.

Use the Ratio Test to determine whether the series

$$\sum_{n=1}^{\infty} \frac{n^2}{3^n} \text{ converges or diverges.}$$

$$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \lim_{n \to \infty} \frac{(n+1)^2}{3^{n+1}} \bigg/ \frac{n^2}{3^n}$$

$$= \lim_{n \to \infty} \frac{(n+1)^2}{3^{n+1}} \cdot \frac{3^n}{n^2} = \lim_{n \to \infty} \left(\frac{n+1}{n}\right)^2 \cdot \frac{1}{3} = \frac{1}{3}$$

The series converges.

Example 2.

Use the RatioTest to determine whether the series

$$\sum_{n=1}^{\infty} \frac{n^8}{(3e)^n} \text{ converges or diverges.}$$

$$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \lim_{n \to \infty} \frac{(n+1)^8}{(3e)^{n+1}} \bigg/ \frac{n^8}{(3e)^n}$$

$$= \lim_{n \to \infty} \frac{(n+1)^8}{(3e)^{n+1}} \cdot \frac{(3e)^n}{n^8} = \lim_{n \to \infty} \left(\frac{n+1}{n}\right)^8 \cdot \frac{1}{3e} = \frac{1}{3e} < 1$$

The series converges.

Root Test:

Example 3.

Use the Root Test to determine whether the series

$$\sum_{n=1}^{\infty} \left(\frac{11n}{n^2 + 15} \right)^n \quad \text{converges or diverges.}$$

We need to check $\lim\limits_{n \to \infty} \sqrt[n]{a_n}$.

$$\lim_{n \to \infty} \sqrt[n]{a_n} = \lim_{n \to \infty} \left(\frac{11n}{n^2 + 15} \right) = 0.$$

The series converges.